

Syllabus Philosophy 3506/5506: Metalogic

Dr. Brian Epstein

In this course, we'll cover the metatheory of propositional and predicate logic, through the Lowenheim-Skolem theorem and Gödel's incompleteness results.

This class is a fairly formal treatment of these topics, and it is very useful to have taken a course in symbolic logic (such as 3505/5505), and to have good mathematical abilities. At the same time, the goal of the class is at least as much to convey the structure and importance of metalogical results, as to develop formal proofs of them.

Required texts:

Hunter, G. *Metalogic: An Introduction to the Metatheory of Standard First Order Logic* (Berkeley, CA: University of California Press, 1973)

Franzen, T. *Gödel's Theorem: An Incomplete Guide to its Use and Abuse* (A.K. Peters, 2005)

Schedule:

(This schedule is tentative, and will change as the course proceeds. Announcements in class and on blackboard take precedence over this schedule)

Week	Date	Reading (from Hunter, unless noted)	Topics
1	Jan. 20	§§1-6 (pp. 3-13)	Introduction to metalogic, outline of course, formal systems and their interpretations
	Jan. 22	§§7-11 (pp. 13-24)	Effective method, sets and their cardinalities, countable and uncountable sets
2	Jan. 27	§§12-14, Appx. 1 (pp. 24-41)	Cantor's theorem, theorems about infinite sets Homework #1 due in class
	Jan. 29	§§15-18 (pp. 46-56)	Functions, truth functions, the formal language P
3	Feb. 3	§§19-20 (pp. 57-62)	Semantics for P, model theoretic concepts, Craig's interpolation theorem Homework #2 due in class
	Feb. 5		NO CLASS
4	Feb. 10	§§21-22 (pp. 62-76)	Adequacy, the formal system PS
	Feb. 12	§§23-24 (pp. 77-83)	Proof-theoretic concepts, concepts of consistency Homework #3 due in class

5	Feb. 17		Test #1
	Feb. 19	§§26-27 (pp. 84-90)	The deduction theorem, mathematical induction
6	Feb. 24	§§28-30 (pp. 91-96)	Soundness of PS, the concept of semantic completeness, outline of Post's proof
	Feb. 26	§32 (pp. 105-111) (skip §31)	Completeness and the Henkin strategy, Lindenbaum's lemma Homework #4 due in class
7	Mar. 3	§32 (pp. 111-116)	Henkin's proof, semantic completeness and compactness
	Mar. 5	§§33-35 (pp. 116-122)	Syntactic completeness, the decidability of PS, extended interpretations of P
8	Mar. 10		SPRING BREAK
	Mar. 12		SPRING BREAK
9	Mar. 17	§§38-39 (pp. 137-142) (skip §§36-37)	Q and its semantics Homework #5 due in class
	Mar. 19	§39 (pp. 142-152)	Model theoretic concepts for Q
10	Mar. 24	§40 (pp. 152-160) (skip p. 160-166)	Model-theoretic metatheorems for Q Homework #6 due in class
	Mar. 26		Test #2
11	Mar. 31	§§41-43 (pp. 166-173)	QS and metatheorems
	Apr. 2	§§44-45 (pp. 173-177)	First order theories, Negation completeness, outline of metatheorems to come
12	Apr. 7	§§45 (pp. 177-180)	Lindenbaum's lemma for first-order theories, extending first-order theories Homework #7 due in class
	Apr. 9	§§45 (pp. 180-185)	Denumerable models
13	Apr. 14	§§45 (pp. 185-190)	Löwenheim-Skolem theorem, compactness Homework #8 due in class
	Apr. 16	§§46, 48, 49 (pp. 195-196, pp. 201-208)	Semantic completeness of QS, non-standard models and their philosophical implications

14	Apr. 21	Franzen Ch. 1-2.6 (pp. 1-39); Appendix A.1 and A.2 (pp. 155-159)	Intro to the incompleteness theorems; Gödel numbering and representability Homework #9 due in class
	Apr. 23	Franzen Ch. 2.7 (pp. 39-49) and handouts	Proving the theorems
15	Apr. 28	Franzen Ch. 2.8-2.10, Ch. 3 (pp. 50-76)	Computability Homework #10 due in class
	Apr. 30	Franzen, Chs. 4 and 5 (pp. 77-115)	Philosophical implications and the limits of the incompleteness theorems
16	May 5		Test #3

Grades:

Homework: 40%

Tests: 60%

Homework:

- Homework is a substantial part of your grade, and crucial for learning the material in the class. Do it, and do it on time!
- If you have a conflict the day before a homework is due, get it done early. Late homework will be marked down 10% per day, weekends and holidays included. I.e., if homework is turned in 1 day late, the highest possible grade is 90%, 2 days late, 80%, and so on.
- Your lowest homework grade will be dropped.
- You are *encouraged* to work on the homework in pairs or small groups. But you **must** write up/submit your homework independently.

Blackboard:

- Announcements and homework assignments will be posted on Blackboard. (www.learn.vt.edu).
- There is also a discussion board on Blackboard, which the TA and I will visit frequently.
- You should expect that the discussion board will be more responsive than emailing us directly, particularly on issues that pertain to course material. Before emailing me or the TA, decide if you can post the question to the discussion threads, so that other people can benefit from it.

Tests / exams:

- There will be three tests over the course of the semester. The final test will concentrate on the last part of the course, but I reserve the right to include material from earlier parts of the course.
- You must take all the exams in order to pass the course.
- Don't skip, forget, sleep through, or otherwise miss the test; you will not be allowed to make it up. Also, don't ask to postpone the test unless there's a true emergency. Failing to be prepared for the test doesn't constitute an emergency.
- The only makeup tests will be in case of a documented illness or other emergency. If you are sick, let me know in advance unless it is a sudden emergency. In any case, medical or other documentation is required to allow for a makeup.

Policies:

- Attendance is required.
- You are expected to have read the assigned material for that day.
- The schedule may change, so check the website for announcements.
- You are expected to abide by the Honor Code.

The Honor Code is the University policy which expressly forbids the following academic violations:

- Cheating -- Cheating includes the actual giving or receiving of any unauthorized aid or assistance or the actual giving or receiving of any unfair advantage on any form of academic work, or attempts thereof.
- Plagiarism -- Plagiarism includes the copying of the language, structure, ideas and/or thoughts of another and passing off same as one's own, original work, or attempts thereof.
- Falsification -- Falsification includes the statement of any untruth, either verbally or in writing, with respect to any circumstances relevant to one's academic work, or attempts thereof. Such acts include, but are not limited to, the forgery of official signatures, tampering with official records, fraudulently adding or deleting information on academic documents such as add/drop requests, or fraudulently changing an examination or other academic work after the testing period or due date of the assignment.